Fractal Tomlinson model for mesoscopic friction: From microscopic velocity-dependent damping to macroscopic Coulomb friction

A. E. Filippov[1] and V. L. Popov[2]
[1]*Donetsk Institute for Physics and Engineering NASU, 83114, Donetsk, Ukraine*
[2]*Technische Universität Berlin, Straße des 17, Juni 135, D-10623 Berlin, Germany*
(Received 19 October 2006; published 15 February 2007)

A modified Tomlinson equation with fractal potential is studied. The effective potential is numerically generated and its mesoscopic structure is gradually adjusted to different scales by a number of Fourier modes. It is shown that with the change of scale the intensity of velocity-dependent damping in an effective Langevin equation can be gradually substituted by an equivalent constant "dry friction." For smooth macrosopic surfaces the effective equation completely reduces to the well known Coulomb law.

DOI: 10.1103/PhysRevE.75.027103 PACS number(s): 81.40.Pq, 64.60.Ak, 68.35.Af

INTRODUCTION

The most striking and famous feature of dry (*Coulomb*) friction is that it is usually almost independent of the sliding velocity [1]. This is in contrast to the linear velocity dependence of microscopic dissipative forces stemming from electron or phonon scattering processes. This qualitative difference of dissipative forces observed on the micro- and macroscales is only one manifestation of a more general phenomenon of the scale dependence of friction: the friction force is indeed nothing but an averaged result of processes occurring on scales which are considered as "microscopic" in a particular problem. The scale dependence of friction is a fundamental theoretical problem of dynamics of systems with scale invariant (fractal) structure and has important practical applications, e.g., in micromechanical and ultrasound devices. Recently developed experimental tools of nanotribology allowed for the investigations of friction at nanometer length scales and stimulated a great increase in the study of friction on the microscopic level during the last decade [2–14]. Many advances have been made in understanding the relationship between macroscopic frictional forces and the microscopic properties of systems [2,8,9,15]. The problem of transition from velocity-depending damping in the microscopic Langevin-type Tomlinson equation [16] to the constant Coulomb friction force has been widely discussed, but its principally mesoscopic (fractal) meaning, which involves a solution of multiscale problem, has not been recognized. The Tomlinson model has been proven to give a good basis to describe tribological experiments using a surface forces apparatus, which moves and interacts on the scales of the atomic potential [10–12]. However, it is certainly not enough when we try to extend the study into meso- and macroscales. In this Brief Report we introduce a fractal description, which extends commonly applied Tomlinson modeling to arbitrary scales in a different phenomenological approach. It provides a "microscopic derivation" of a relatively simple but yet useful equation of the Tomlinson type. This equation appears to describe intermediate dynamical regimes in the whole interval from nanoscale to the macroscopic Coulomb friction. We propose a simple experimental procedure which allows obtaining phenomenological parameters of the equation.

MODEL AND DISCUSSION

Let us start from the commonly used Tomlinson model [16] that has proven to be successful in describing shear response in surface forces apparatus (SFA) configuration, namely the lateral motion of a driven plate,

$$M\partial^2 x/\partial t^2 + \gamma \partial x/\partial t + \partial U(x)/\partial x + K(x - Vt) = 0. \quad (1)$$

Here a driven plate of mass M and the center-of-mass coordinate x is pulled by a spring of a spring constant K. In standard Tomlinson equation $U(x)$ is the effective periodic potential $U(x)=U_0\cos(2\pi x/b)$ experienced by the plate due to the presence of an embedded system. Below for general theoretical study we use nondimensional units normalized to characteristic microscopic scales and energies of a particular system. The parameter γ is responsible for dissipation. The spring is connected to a stage which moves with velocity V. This equation was recently supported microscopically, but was never extended even to the closest mesoscopic scales. The main problem is that the mesoscopic structure of frictional surfaces is of a fractal character [17] and thereby cannot be characterized by a certain wave vector (or even few ones [2,3,8,9]) like it is normally used in applications of the Tomlinson model. Below we extend the model into a fractal potential.

Let us consider a fractal potential [17] of the form

$$U(x) \equiv U_{fractal}(x) = U_0 \int_{q_1}^{q_2} dq c(q)\cos(qx+\zeta), \quad c(q) = q^{-\beta}. \quad (2)$$

Here q_1 and q_2 are characteristic cut off wave vectors and $\zeta(x)$ is a random phase that we assume δ correlated $\langle\zeta(q)\zeta(q')\rangle = 2\pi\delta(q-q')$. For the majority of physically interesting systems index β is close to $\beta \approx 0.9$ [17,18]. Below we will keep this value for definiteness. For further study it is convenient to go over to a discrete representation of the integral in Eq. (2) $\int dqc(q) \to \Sigma$ with a discrete step Δq between the wave vectors determined by the smallest vector q_1 corresponding to an inverse maximal length l_{max} of the system which equals normally its size $l_{max}=L$. The total number N_{tot} of the terms in the sum is given by $N_{tot}=q_2/q_1 \equiv q_2/\Delta q$. A discrete approach is natural for further numerical

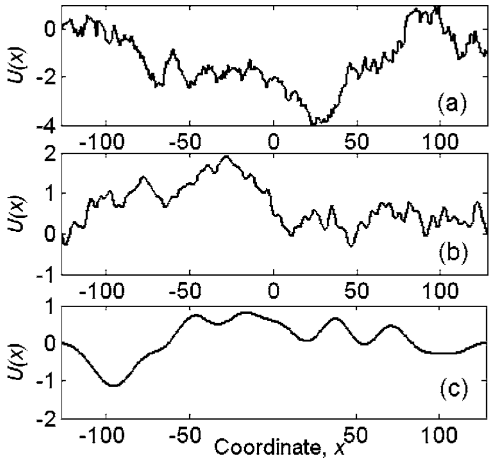

FIG. 1. Fragments of fractal potential generated at different number of modes included: (a) $N_{modes}=256$; (b) $N_{modes}=64$; (c) $N_{modes}=8$.

study. It allows us to adjust the potential to different scales by the number of the modes included into summation $N_{modes}<N_{tot}$.

Normally the Tomlinson model is based on an analytical definition of the potential. The force is calculated at each time step and the procedure can be formally extended to infinite time-space runs. The modified fractal model operates with the data array $U_{fractal}(x)$. So, its generation has to be extended to an infinite run too. For this sake, instead of Eq. (2) we use the following differential definition of fractal potential: $\partial U_{fractal}(x)/\partial x = U_0 \Delta x \Sigma_j q_j c(q_j)\sin(q_j x+\xi)$; where $j=1,2,\ldots N_{modes}$. It allows us to extend $U_{fractal}(x)$ infinitely each time, when the x coordinate runs out of the array bonds. For numerical procedure it means that the modified Tomlinson equation

$$\partial^2 x/\partial t^2 + \gamma \partial x/\partial t + \partial U_{fractal}(x)/\partial x + K(x-Vt) = 0 \qquad (3)$$

is actually extended by an additional differential equation which is solved in parallel.

To study the scale dependence of friction force, we generate a set of fractal potentials for a different number of modes included ($N_{modes}<N_{tot}$). This number defines a cutoff wave vector $q_{cutoff}=\Delta q N_{\text{modes}}$ and the corresponding cutoff wave length $\lambda_{cutoff}=2\pi/q_{cutoff}$ of the potential. Some of the potentials with different N_{modes} are shown in Fig. 1. All of them correspond to the same potential "measured" with different spatial resolution. All space scales larger than the cutoff wavelength are included in the potential $U_{fractal}(x)$ and should be treated explicitly in the frame of the dynamic model like Eq. (3).

Typical time dependence of the friction force is presented in Fig. 2. The initial short fragment of the dependence is shown in the inset to the figure. It resembles a standard stick-slip behavior of the friction in Tomlinson model with periodic potential [11]. The only difference is a variation of the stick-slip oscillations due to randomness of the potential. Long-time behavior has horizontal asymptotics (in average) which is defined by the product of damping constant γ and driving velocity V. To get the mean friction at a given number of modes and velocities one has to omit the initial transient interval and average over the long-time asymptotic part. The results of the calculations are accumulated in Fig. 3.

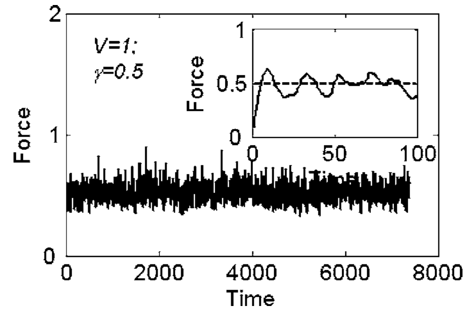

FIG. 2. Typical long-time dependence of friction force for fractal system (at $V=1$ and $\gamma=0.5$). Short initial part of the dependence with initial transient interval is shown in the inset.

As it was expected at large velocities $V\to\infty$ the friction force is proportional to V and degenerates into constant friction $F_{eff}>0$ at low velocities $V\to 0$. But, the dependence is different for different N_{modes}. Let us note that this difference is found explicitly in numerical experiment where one can keep a desirable number of modes. In reality some microscopic space scales with wavelengths $\lambda<\lambda_{cutoff}$ cannot be treated explicitly. Our intention is to describe them by an additional "friction" force taking into account all the processes occurring on the microscopic scales. This means that if we use a smoothed fractal potential in Eq. (3) instead of the exact one, we should add an additional friction force accounting for microscopic processes. This friction force compensating the excluded microscopic modes will be scale dependent. Below we search for a compensating force of the form $\gamma_{eff}v+F_{eff}$. Thus on a mesoscale we propose the following phenomenological equation instead of Eq. (3):

$$\partial^2\tilde{x}/\partial t^2 + \gamma_{eff}(N_{modes},\partial\tilde{x}/\partial t)\cdot\partial\tilde{x}/\partial t + F_{eff}(N_{modes}) + K(\tilde{x}-Vt) + \partial\tilde{U}_{fractal}(\tilde{x},N_{modes})/\partial\tilde{x} = 0. \qquad (4)$$

Here all variables marked with "~" are smoothed variables

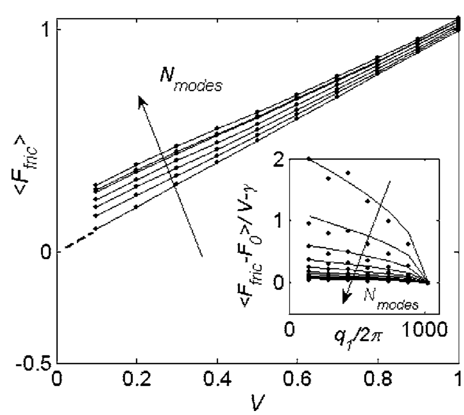

FIG. 3. Mean friction force at different number of modes included to the potential as a function of velocity V. Inset illustrates a procedure of finding velocity dependence of the renormalized damping constant described in the text.

of the mesocopic scale corresponding to the truncation of the fractal potential. We expect that γ_{eff} will achieve the maximum value γ and F_{eff} the value $F_{eff}=0$ if all potential modes are included, i.e., in the "truly microscopic limit." In the other limit $N_{modes}=0$ we expect to see a pure "macroscopic friction force" F_{eff} with a diminished damping constant γ_{eff}. Based on results of numerical experiments with the model Eq. (4) we found that the following ansatz keeps the macroscopic friction force invariant to the number of modes included in the model:

$$\gamma_{eff}(N_{modes},\partial\tilde{x}/\partial t) = \gamma\{1 - c_1(1 - N_{modes}/N_{\max})^{1/2}/[1 + c_2(\partial\tilde{x}/\partial t)^2]\};$$

$$F_{eff}(N_{modes}) = F_0[1 - (1 - N_{modes}/N_{\max})], \qquad (5)$$

where $F_0 \approx c_1 \approx 0.25$; $c_2 \approx 0.4$ for $\gamma=1$. The inset to Fig. 3 illustrates a procedure which leads to the ansatz Eq. (5). In the procedure we calculate actual friction force $F_{fric}=K(\tilde{x}-Vt)$ (counted from its minimum value in the array $F_0 = F_{fric}|_{N_{\mathrm{modes}}=N_{\max}}$), divide it by the velocity V, and find the difference between this combination and trial damping constant γ. Equation (5) gives good interpolation for the resulting array. Corresponding curves are shown by the lines in the inset to Fig. 3. The renormalized mesoscopic friction force for different truncations of the potential calculated from Eq. (4) is presented in Fig. 4. The resulting force does not depend on the number of modes included in the potential. It proves that Eqs. (4) and (5) provide desirable universal description of the friction in the mesoscopic scale.

The ansatz Eq. (5) does not have direct physical meaning, but *a posteriori* one can give it the following qualitative treatment. The effective friction force degenerates into static dry friction for a macroscopically flat surface when the number of modes included in the potential goes to zero, $N_{modes}=0$. In its turn, the velocity dependent part of the friction returns to its microscopic value either in the opposite limit when N_{modes} is maximal, or at high absolute velocities.

Current experimental resolution admits a direct check of the results obtained. One can measure time-dependent friction force with high accuracy (a few nanometers per second) and gradually exclude fast harmonics by filtering of the time series over appropriate scales. Such a study has been performed by our experimental group in Berlin TU. It was found that static and velocity dependence impacts the friction change according to the above expectations, and scale dependence can be eliminated according to the scheme Eqs. (4) and (5).

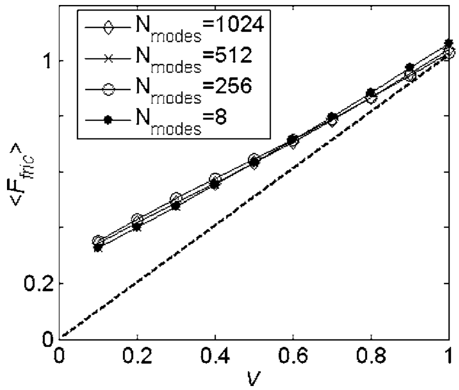

FIG. 4. Mean friction force at different N_{modes} as a function of velocity V, obtained using renormalized equation.

CONCLUSION

To summarize, we have proposed a model equation that establishes relationships between mesoscopic frictional phenomena and microscopic dynamics of velocity-dependent damping. The model involves random potential with fractal spectrum that depends on the scale of the problem. It leads to a derivation of an effective equation of the Tomlinson type. This equation incorporates both static and velocity dependent impacts to the friction and appears to describe intermediate dynamical regimes in the whole interval from nanoscale to the macroscopic Coulomb friction. An experimental procedure which allows obtaining phenomenological parameters of the equation is proposed.

ACKNOWLEDGMENTS

One of the authors (A.F.) is grateful to the German Research Society and European Science Foundation for financial support during his stay at Berlin Technological University.

[1] B. N. J. Persson, *Sliding Friction, Physical Properties and Applications* (Springer, Berlin, 2000).
[2] M. G. Rozman, M. Urbakh, and J. Klafter, Phys. Rev. Lett. **77**, 683 (1996).
[3] V. Zaloj, M. Urbakh, and J. Klafter, Phys. Rev. Lett. **81**, 1227 (1998).
[4] M. Weiss and F. J. Elmer, Phys. Rev. B **53**, 7539 (1996).
[5] V. Zaloj, M. Urbakh, and J. Klafter, Phys. Rev. Lett. **82**, 4823 (1999); H. G. E. Hentschel, F. Family, and Y. Braiman, *ibid.* **83**, 104 (1999).
[6] M. Porto, M. Urbakh, and J. Klafter, Europhys. Lett. **50**, 326 (2000).
[7] E. Gnecco *et al.*, Phys. Rev. Lett. **84**, 1172 (2000).
[8] M. H. Muser, L. Wenning, and M. O. Robbins, Phys. Rev. Lett. **86**, 1295 (2001).
[9] M. H. Muser, Phys. Rev. Lett. **89**, 224301 (2002).
[10] A. E. Filippov, J. Klafter, and M. Urbakh, Phys. Rev. Lett. **92**, 135503 (2004).
[11] A. E. Filippov, J. Klafter, and M. Urbakh, Phys. Rev. Lett. **87**, 275506 (2001).
[12] Z. Tshiprut, A. E. Filippov, and M. Urbakh, Phys. Rev. Lett. **95**, 016101 (2005).

[13] C. Daly, J. Zhang, and J. B. Sokoloff, Phys. Rev. Lett. **90**, 246101 (2003).
[14] A. Socoliuc *et al.*, Phys. Rev. Lett. **92**, 134301 (2004).
[15] H. Risken, *The Fokker-Planck Equation* (Springer, Berlin, 1996).
[16] G. A. Tomlinson, Philos. Mag. **7**, 905 (1929).
[17] V. L. Popov and Ya. Starcevic, Tech. Phys. Lett. **31**, 309 (2005).
[18] O. K. Dudko, V. L. Popov, and G. Putzar, Tribol. Int. **39**, 456 (2006).