Kinetics of vortex formation in superconductors with *d* pairing

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We study the kinetics of vortex formation in superconductors with *d* pairing. We find order-parameter peculiarities and associated magnetic field maxima at intermediate stages of the evolution from the disordered to the ordered state. [S0163-1829(96)09926-2]

INTRODUCTION

Traditionally, heavy-fermion systems are treated as systems with *d* pairing. Symmetry analysis gives the same superconducting class for Y-Ba-Cu-O.[1] Recent experiments[2] and theoretical studies[3] provide extensive evidence of this fact for the high-T_c superconductors. It has been found that these systems are type-II superconductors and they should possess Abrikosov-like vortices. Static vortex structures have been found experimentally in these systems. As a rule, these structures may be described in the framework of a modified theory of the vortex state, accounting for their strong anisotropy, interaction between superconducting layers, quasi-two-dimensional (quasi-2D) behavior, and the possible existence of the multiquantum vortices.[3]

However, the formation of this state in kinetics may be very nontrivial. A vortex (more exactly, a pair of them: ''vortex-antivortex'') is an essentially mesoscopic structure. It forms as a dissipative attractor during a kinetic process[4] when the fluctuations of both components of the order-parameter field ψ_j interact with the (gauge) electromagnetic field A.

For a traditional superconductor the order parameter ψ is a two-component vector. It seems essential in our context, because it leads automatically to the necessary topological configurations from arbitrary fluctuations.[4,5] In short, the vortices arise at the points where the two lines $\psi_1=0$ and $\psi_2=0$ intersect. The existence of such an intersection is typical for a two-component field. Formally, if the number of components is larger than $n=2$, the probability of mutual intersection of all lines $\psi_j=0$ at the same (unique) point is extremely small. In principle, this may suppress the vortex generation.

However, as mentioned above, the existence of the vortex state in high-T_c superconductors is well established. In the above context two explanations are possible: (1) the concept of *d* pairing is not valid for these systems; or, (2) some kinetic mechanism exists which stably produces the vortices. We concentrate on this problem in the present paper.

MODEL AND RESULTS

The Ginzburg-Landau-Wilson (GLW) functional for *d* pairing may be written in the form[1,6–9]

$$\begin{aligned}H=1/2\int dV\{&\alpha\eta\eta^*+\beta_1(\eta\eta^*)^2/2+\beta_2|\eta\cdot\eta|^2/2\\&+\beta_3[|\eta_x|^4+|\eta_y|^4]/2+K_1D_i^*\eta_j^*D_i\eta_j+K_2D_i^*\eta_i^*D_j\eta_j\\&+K_3D_i^*\eta_j^*D_j\eta_i+K_4D_z^*\eta_i^*D_z\eta_i+\gamma(\mathrm{rot}A)^2\},\end{aligned}\tag{1}$$

where $D_i=\partial_i-igA_i$, $j=x,y$; A is a vector potential; $\eta=\{\eta_x,\eta_y\}, \eta_x=\eta_1+i\eta_2, \eta_y=\eta_3+i\eta_4$; $a=\alpha(T-T_c)$; β_1,β_2,β_3 and K_i are phenomenological constants; $\gamma=1/8\pi$, and $g=2e/\hbar c$. Magnetic stability and the positiveness of the quartic form in the functional (1) lead to the following restrictions:

$$\begin{gathered}K_1>|K_2|,\quad K_1+K_2+K_3>|K_2|,\quad K_4>0,\\\beta_1+\beta_2+\beta_3/2+\min[\beta_3/2;-(\beta_2+|\beta_2|)/2]\geqslant 0.\end{gathered}\tag{2}$$

Let us consider the specific situations existing in UPt_3 and Y-Ba-Cu-O systems, where the coefficient β_3 is anomalously small and $\beta_1\approx\beta_2$; $\beta_2>0$.[10,11] Under these conditions and in the absence of an external magnetic field, an ordered state with broken time-inversion invariance occurs. Below, for simplicity, we suppose that $\beta_3=0$. This simplifies all equations, but changes the free-energy functional from square to cylindrical symmetry. It may be proven that a small correction from $\beta_3\neq 0$ (but at $\beta_3\ll\beta_1,\beta_2$) leads to some renormalization of the parameters, but does not strongly change the general physical picture. When $\beta_3=0$ the condition (2) is transformed to a simpler one: $\beta_1>0$, $\beta_1>-\beta_2$.

In the presence of fluctuation noise $f(r,t)$, the evolution of the order parameter may be described on the basis of the well-known modification of the Landau-Khalatnikov equations in the form

$$\begin{aligned}\partial\eta_i/\partial t&=-\Gamma\,\delta H/\delta\eta_i+f(r,t),\\\partial A/\partial t&=-\Gamma_A\delta H/\delta A+f(r,t),\end{aligned}\tag{3}$$

where Γ and Γ_A are kinetic constants and

$$\langle f(r,t)\rangle=0,\quad\langle f(r,t)f(r,t)\rangle=D\,\delta(r-r')\,\delta(t-t').$$

Taking into account the anisotropy and hierarchy of interactions in the substances under consideration (and for simplicity), we suppose in what follows that no variables depend on the z coordinate. This means that we study numerically a two-dimensional cross section of a real 3D system in the plane perpendicular to the c axis.[4,5] In addition, for the same

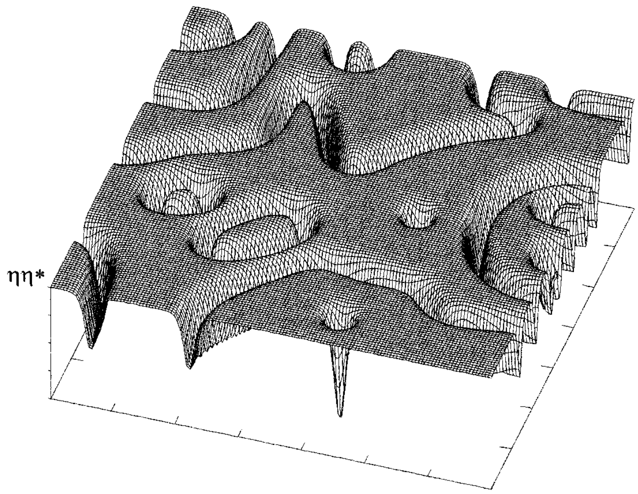

FIG. 1. Fragment of the system at an intermediate stage of evolution containing typical configurations of the order-parameter modulus $S(x,y)$ [calculated at $b=2\beta_2/(\beta_1+\beta_2)=1$].

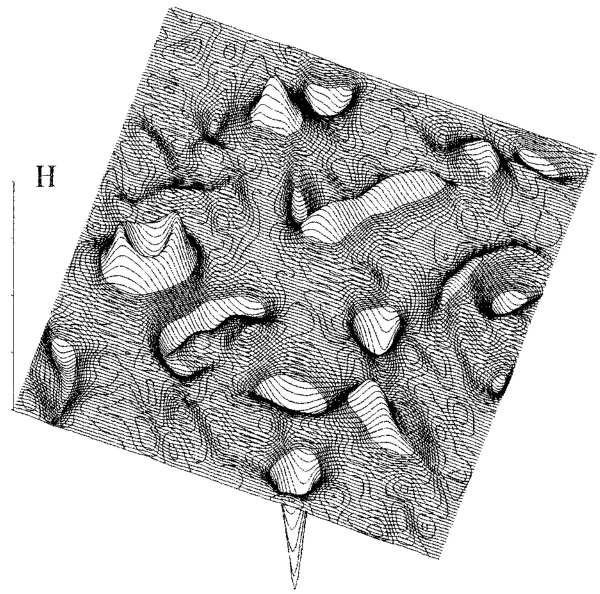

FIG. 2. The same as Fig. 1 for magnetic field h. Mutual correspondence between the distribution of the magnetic field and the structure of $S(x,y)$ is quite visible directly. The ''usual vortices'' presented here are the results of the collapse of the ''channels'' at an earlier stage.

reasons we limit ourselves to the case $K_2,K_3 \ll K_1$. As a result we have the system of equations

$$\partial\eta_j/\partial t=\Delta\eta_j-(-1)^j(2A\nabla\eta_k+\eta_k\nabla A)-\eta_j[A^2-1+S] + bM\partial M/\partial\eta_j+f, \quad [j=1,\ldots,4, k=j-(-1)^j],$$
$$\vartheta_A^{-1}\partial A/\partial t=-\mathrm{rot}(\mathrm{rot}A)+\{[\eta_1\nabla\eta_2-\eta_2\nabla\eta_1] + [\eta_3\nabla\eta_4-\eta_4\nabla\eta_3]-AS\}/\varkappa^2+f. \tag{4}$$

The standard notation $\varkappa$ is used for the Ginzburg-Landau parameter in these dimensionless equations. For typical superconductors with d pairing one has $\varkappa \gg 1$. The following notation is used here:

$$S=\sum \eta_i^2 \equiv \eta\eta^*, \quad M=[\eta_1\eta_4-\eta_2\eta_3]\equiv i[\eta\times\eta^*]/2.$$
$$\eta_i=\eta_i[(\beta_1+\beta_2)/\alpha]^{1/2}, \quad \vartheta_A=(\Gamma^{-1}\Gamma_A/8\pi\alpha\xi^2)\sim 1,$$
$$b=2\beta_2/(\beta_1+\beta_2)<2, \quad \xi=(K_1/\alpha)^{1/2}.$$

Equations (4) were solved by the numerical approach already used in Refs. 5 and 12. Figures 1 and 2, respectively, show a typical fragment of the (x,y)-space distributions of the order-parameter modulus $S=\Sigma\eta_i^2$ and magnetic field $h=\mathrm{rot}A$, at an intermediate stage of evolution. To calculate them, initial conditions with zero order parameter and boundary conditions corresponding to zero external magnetic field were used.

Two types of the peculiarities presenting in the system are quite visible on the pictures. They are the ''usual vortices'' and specific ''channels'' on the $\eta(x,y)$ surface, with a width $\Delta=[\xi^2(\beta_1+\beta_2)/\beta_2]^{1/2}$ and having magnetic field inside them. Generation of the ''channels'' is an interesting feature of the system under consideration. At intermediate steps of the evolution these ''channels'' form very complicated configurations. During the relaxation of the system to the ordered state these ''channels'' take the simpler form of rings. After that, there are two different possibilities: (1) the ring ''channel'' completely disappears at $t\to\infty$; (2) it transforms to the ''usual'' vortex.

Figure 3 reproduces the typical stages of this transformation corresponding to the second case. In that case the initial ring has a ''topological charge'' corresponding to the vortex from the beginning. The ''usual'' vortices displayed in Figs. 1 and 2 are the results of this process. The only ''channels'' which are pinned on the boundary (for example, on the boundaries of granules) do not disappear.

Numerical observation of excitations stimulates the search for an approximate analytical solution for them. It is easy to see that at least in the regions with small curvature, a good approximation for the ''channels'' may be obtained even from the one-dimensional version of the equations.

When the parameter $\varkappa \gg 1$ one can prove that the gauge field A does not strongly effect the η_j distributions. So, to calculate them in lowest approximation it is quite sufficient to study the following reduced system:

$$d^2S/dx^2-2\left[\sum_j (d\eta_j/dx)^2\right]-2S(S-1)+4bM^2=0, \tag{5a}$$

$$d^2M/dx^2-2[(d\eta_1/dx)(d\eta_4/dx)-(d\eta_2/dx)(d\eta_3/dx)] -2M(S-1)+bSM=0. \tag{5b}$$

After that one can determine the field h, if it is necessary. However, even in the last form this system is very complicated. We utilized the method of verified smallness used recently in Ref. 5. The concept may be reproduced as follows.

First of all, we solve the equations numerically and find which terms (or their combinations) in the equations are small in comparison with other terms. Of course, this small-

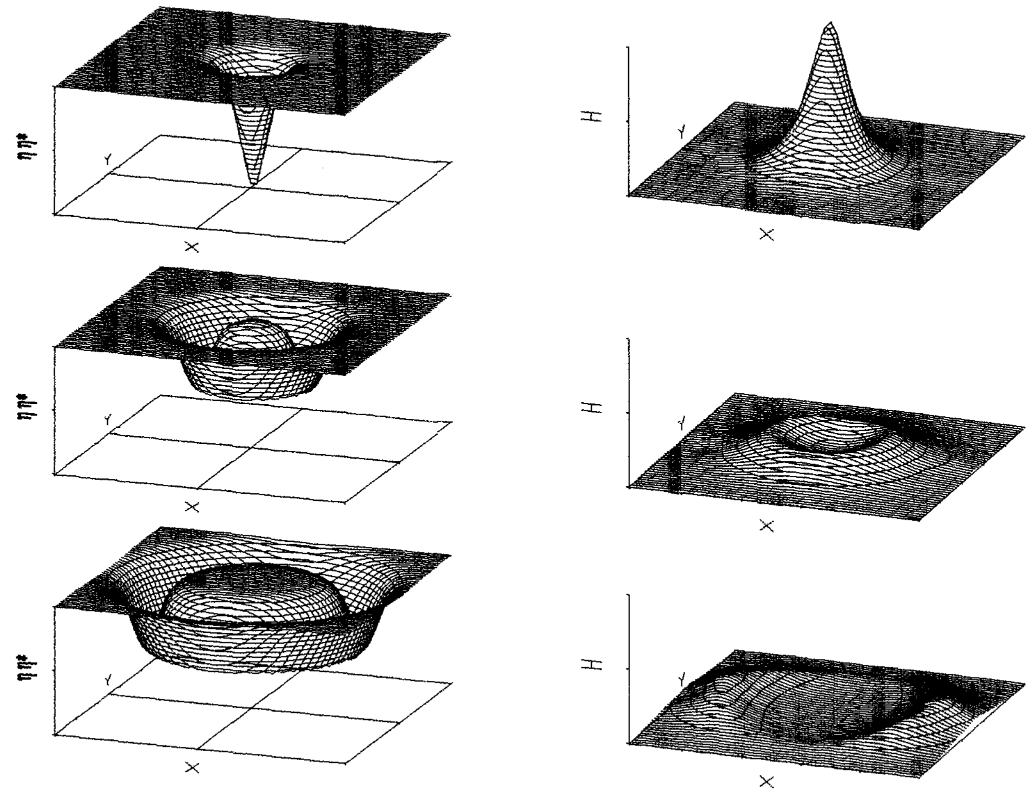

FIG. 3. Typical stages of transformation of the ringlike structure into the ‘‘usual’’ vortex. Two (left and right) parallel sequences of the plots show order parameter and magnetic field, respectively.

ness should occur in quite a wide interval of the parameters. In particular, we found that the terms in square brackets in Eqs. (5) may be omitted. As a result, we obtain a system having two invariants, S and M, only.

Using different projections of the phase pattern of the system in the space $(S,M,dS/dx,dM/dx)$, one may establish an approximate relation between these two variables in the region of the ‘‘channel.’’ As a result, a unique equation remains which may be solved directly.

This procedure has been performed and the approximate relation $S(M)\simeq 1+b(2-b)M^2$ has been found. In reality, it is satisfied with a quite good accuracy (up to 1%). Substituting this relation into the system (5) one has a simple equation for M:

$$d^2M/dx^2+bM-b(2-b)^2M^3=0.$$

For the boundary conditions $x\rightarrow-\infty$, $M\rightarrow -M_0 \equiv -1/(2-b)$ and $x\rightarrow+\infty$, $M\rightarrow +M_0$ it has a well-known solution:

$$M(x)=[\eta_1\eta_4-\eta_2\eta_3]=Q\tanh(x/\Delta). \tag{6}$$

Here, $\Delta=(2/b)^{1/2}=[\xi^2(\beta_1+\beta_2)/\beta_2]^{1/2}$ is the width of the wall, and $Q=(M_{+\infty}-M_{-\infty})/2$ is the ‘‘topological charge.’’ For $S(x)$ one has an analytical approximation in the form of the ‘‘dark soliton’’:

$$S(x)=\sum_{i=1}^{4}\eta_i^2=1+b[\tanh(x/\Delta)]^2/(2-b). \tag{7}$$

It has been found that (at $0<b<1.85$) the difference between these estimations and the numerical results is smaller than the level of the noise.

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